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Criteria interactions in multiple criteria decision aiding: A Choquet formulation for the TODIM method

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Abstract

The problem of modeling interactions between criteria in Multiple Criteria Decision Aiding (MCDA) has been approached by a number of authors. That problem arises when there are visible redundancies and synergies among criteria. Aggregation models exist that have been conceived to take care of criteria interdependences. From these aggregation models a special attention has been given to the Choquet integral, and its possibilities and limitations have been pointed out. In this paper it is shown how measures of criteria interaction can be computed for the TODIM method of MCDA by using the Choquet integral. The key conclusions from this case study are listed below: (i) the use of the Choquet integral minimizes the calculations by TODIM since it is unnecessary to normalize the raw data; (ii) not only precise values can be used but also interval data; this second situation would lead to using a fuzzy triangular number; (iii) by using the Choquet integral more complex additive models can be used that allow for taking dependencies between criteria into consideration. As important elements of future research the authors plan to encompass in their analysis new concepts such as generalizations of Choquet integral and the bipolar Cumulative Prospect Theory.

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Keywords: Prospect theory; Real estate; Fuzzification; Additive value function

1. Introduction

The problem of modeling interactions between criteria in Multiple Criteria Decision Aiding (MCDA) has been approached by a number of authors. That problem arises when there are visible redundancies and synergies among criteria as it was clearly stated by [1], who analyzed available aggregation models that have been conceived to take care of criteria interdependences. From these aggregation models a special attention was then given to the use of the Choquet integral, by pointing out its possibilities and limitations. More recently, that problem has been tackled by Robust Ordinal Regression (ROR) [2]. Similarly, ROR has also been applied to take into account imprecise evaluations in MCDA [3].

In this paper it is shown how measures of criteria interaction can be computed for the TODIM method of MCDA. This is a method that relies on an additive value function built having as a basis the paradigm of prospect theory [4], [5]. The TODIM method was formulated in the early 1990s and has been the object of a number of publications

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since then [6], [7], [8], [9] and [10]. TODIM is based on nonlinear prospect theory [11] as the shape of its value function is the same as the gains/losses function of prospect theory. Gains and losses in prospect theory are always referred to a reference point. The mathematical formulation of the TODIM method follows. Mathematical expressions (1), (2) and (3) constitute the modeling underlying the use of the TODIM method:

$$\delta(A_i, A_j) = \sum_{c=1}^{m} \Phi_c(A_i, A_j) \qquad i, j = 1,\dots,n \tag{1}$$

$$\Phi_c(A_i, A_j) = \begin{cases} \sqrt{\dfrac{w_{rc}(P_{ic} - P_{jc})}{\sum_{c=1}^{m} w_{rc}}} & if\,(P_{ic} - P_{jc}) > 0 \\ 0 & if\,(P_{ic} - P_{jc}) = 0 \\ -\dfrac{1}{\theta}\sqrt{\dfrac{(\sum_{c=1}^{m} w_{rc})(P_{jc} - P_{ic})}{w_{rc}}} & if\,(P_{ic} - P_{jc}) < 0 \end{cases} \tag{2}$$

$$\xi_i = \frac{\sum_{j=1}^{n} \delta(A_i, A_j) - \min \sum_{j=1}^{n} \delta(A_i, A_j)}{\max \sum_{j=1}^{n} \delta(A_i, A_j) - \min \sum_{j=1}^{n} \delta(A_i, A_j)} \tag{3}$$

with the following parameters and variables:
$\delta\ (A_i, A_j)$, measurement of dominance of alternative A_i over alternative A_j;
n, number of alternatives;
m, number of criteria;
c, a generic criterion;
w_{rc}, trade-off rate (or trade-off weighting factor) between the reference criterion *r* and any other, generic criterion *c*;
P_{ic}, P_{jc}, evaluations of alternatives *i* and *j* with respect to criterion *c*;
θ, attenuation factor of the losses, different choices of θ lead to different shapes of the prospect theoretical value function in the negative quadrant;
$\Phi_c(A_i, A_j)$, contribution of criterion c to function $\delta(A_i, A_j)$, when comparing alternatives A_i and A_j.
ξ_i, normalized global performance of alternative A_i, when compared against all other alternatives.

The extension of the original formulation of TODIM towards a Cumulative Prospect Theory (CPT) [12] interpretation of this method was presented by [13]. In the next section we show how the TODIM method can be extended by applying the unipolar Choquet integral [14], [15] and how this extension allows computing measures of interaction between criteria.

2. The Choquet-extended TODIM method

From the original formulation of TODIM we compute the measure of relative dominance of each alternative A_i over another alternative A_j as equation (1), repeated below:

$$\delta(A_i, A_j) = \sum_{c=1}^{m} \Phi_c(A_i, A_j) \qquad i, j = 1,\dots,n \tag{1}$$

Through considering the fuzzy measures μ of interactions between criteria we can obtain the overall value of each alternative with no need of normalization. This is accomplished by rewriting (1) as equation (4):

$$\delta(A_i, A_j) = I_\mu(a)\Phi_c(A_i, A_j) \tag{4}$$

where a: S →R , and I is the Choquet integral in relation to the fuzzy measure μ. Suppose that criteria are ordered as follow: $C_1 > C_2 > ... > C_m$. We can now determine the fuzzy measures (i.e., the criteria interactions) as in equation (5):

$$\mu_1 = k_1;\ \mu_{12} = k_2\mu_1; .....\mu_{j-1,j} = k_j\mu_{j-1}, \sum_{j=1}^{m} k_j\mu_j = 1 \tag{5}$$

where k_j are constants. The performance matrix can now be rewritten as follows in Table 1.

Table 1: Performance matrix

Criteria	Alternatives			
	A_1	A_2	. . .	A_n
C_1	$\mu_1\Phi(A_1,C_1)$	$\mu_1\Phi(A_2,C_1)$	. . .	$\mu_1\Phi(A_n,C_1)$
C_2	$\mu_{12}\Phi(A_1,C_2)$	$\mu_{12}\Phi(A_2,C_2)$	. . .	$\mu_{12}\Phi(A_n,C_2)$
. . .	. . .	. . .	. . .	. . .
C_m	$\mu_{m-1,m}\Phi(A_1,C_m)$	$\mu_{m-1,m-1}\Phi(A_1,C_m)$	. . .	$\mu_{m-1,m-1}\Phi(A_n,C_m)$

3. An application case study

The case study is a valuation of residential properties carried out by real estate agents in the city of Volta Redonda, Brazil. Fifteen properties in different neighborhoods were analyzed as alternatives and a total of eight evaluation criteria were identified. A detailed description of the alternatives and criteria can be found in [7]. The initial weights assigned to the criteria used to evaluate the properties were defined by decision makers (i.e., the real estate agents), assigning a number between 1 and 5 to each criterion where 1 would mean 'least important' and 5 would mean 'most important'. The alternatives follow:

A_1 – A house in an average location, with 290 m^2 of constructed area, a high standard of finishing, in a good state of conservation, with one garage space, 6 rooms, a swimming pool, barbecue and other attractions, without a security system.

A_2 – A house in a good location, with 180 m^2 of constructed area, an average standard of finishing, in an average state of conservation, with one garage space, 4 rooms, a backyard and terrace without a security system.

A_3 – A house in an average location, with 347 m^2 of constructed area, a low standard of finishing, in an average state of conservation, two garage spaces, 5 rooms, a large backyard, without a security system.

A_4 – A house in an average location, with 124 m^2 of constructed area, an average standard of finishing, in a good state of conservation, two garage spaces, 5 rooms, a fruit orchard, a swimming pool and barbecue, without security system.

A_5 – A house in an excellent location, with 360 m^2 of constructed area, a high standard of finishing, in a very good state of conservation, four garage spaces, 9 rooms, a backyard and manned security boxes in the neighborhood streets.

A_6 – A house located between the periphery and the city center (periphery/average location) with 89 m^2 of constructed area, an average standard of finishing, in a good state of conservation, with one garage space, 5 rooms, a backyard, without a security system.

A_7 – An apartment located in the periphery, with 85 m^2 of constructed area, a low standard of finishing, in a bad state of conservation, one garage space, 4 rooms, a manned entrance hall with security.

A_8 – An apartment in an excellent location, with 80 m^2 of constructed area, average standard of finishing, good state of conservation, with one garage space, 6 rooms, manned entrance hall with security.

A_9 – An apartment located between the periphery and the city center (periphery/average location), with 121 m^2 of constructed area, an average standard of finishing, in a good state of conservation, no garage space, 6 rooms, without a security system.

A_{10} – A house located between the periphery and the city center (periphery/average location), with 120 m^2 of constructed area, a low standard of finishing, in a good state of conservation, with one garage space, 5 rooms, a large backyard, without a security system.

A_{11} – A house in a good location, with 280 m^2 of constructed area, an average standard of finishing, in an average state of conservation, with two garage spaces, 7 rooms, with an additional security system.
A_{12} – An apartment located in the periphery, with 90 m^2 of constructed area, a low standard of finishing, in a bad state of conservation, one garage space, 5 rooms, without additional security.
A_{13} – An apartment located in the periphery in an average location, with 160 m^2 of constructed area, a high standard of finishing, in a good state of conservation, two garage spaces, 6 rooms, with additional security features.
A_{14} – An apartment in a good location, with 320 m^2 of constructed area, high standard of finishing, in a good state of conservation, 2 garage spaces, 8 rooms, with in addition a security system.
A_{15} – A house in a good location, with 180 m^2 of constructed area, an average standard of finishing, in a very good state of conservation, one garage space, 6 rooms, with in addition a security system.

Table 2 shows a list and a description of criteria, with their assigned and normalized weights. Table 3 is the evaluation matrix.

Table 2: Criteria weights

Criterion	Description	Assigned weights	Criteria weights
C_1	Localization	5	0.25
C_2	Construction area	3	0.15
C_3	Quality of construction	2	0.1
C_4	State of conservation	4	0.2
C_5	Number of garage spaces	1	0.05
C_6	Number of rooms	2	0.1
C_7	Attractions	1	0.05
C_8	Security	2	0.1

Table 3: Evaluations matrix

Alternative	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	3	290	3	3	1	6	4	0
A_2	4	180	2	2	1	4	2	0
A_3	3	347	1	2	2	5	1	0
A_4	3	124	2	3	2	5	4	0
A_5	5	360	3	4	4	9	1	1
A_6	2	89	2	3	1	5	1	0
A_7	1	85	1	1	1	4	0	1
A_8	5	80	2	3	1	6	0	1
A_9	2	121	2	3	0	6	0	0
A_{10}	2	120	1	3	1	5	1	0
A_{11}	4	280	2	2	2	7	3	1
A_{12}	1	90	1	1	1	5	2	0
A_{13}	2	160	3	3	2	6	1	1
A_{14}	3	320	3	3	2	8	2	1
A_{15}	4	180	2	4	1	6	1	1

Computations are performed in 5 steps:

- Step 1: Fuzzification of the scales of criteria in order to become non dimensional

In this case study fuzzy triangular membership functions with null amplitude and mode equal to the original scale are used. Those fuzzy triangular membership functions are written as equation (6) below:

$$f(x,b,c,d) = \max(\min(\frac{x-b}{c-b};\frac{d-x}{d-c}),0) \tag{6}$$

b, c, d are parameters. Parameters b and c locate the base of the triangle and parameter d locates the vertex. Table 4 shows the evaluation matrix obtained after fuzzification.

Table 4: The evaluation matrix can now be rewritten after accomplishing the fuzzification

Alternative	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	0.040	0.053	0.053	0.053	0.027	0.027	0.067	0.013
A_2	0.053	0.040	0.040	0.040	0.027	0.007	0.040	0.013

A_3	0.040	0.067	0.016	0.040	0.040	0.013	0.027	0.013
A_4	0.040	0.027	0.040	0.053	0.040	0.013	0.067	0.013
A_5	0.067	0.067	0.053	0.067	0.067	0.067	0.027	0.067
A_6	0.027	0.013	0.040	0.053	0.027	0.013	0.027	0.013
A_7	0.013	0.013	0.016	0.027	0.027	0.007	0.013	0.067
A_8	0.067	0.013	0.040	0.053	0.027	0.027	0.013	0.067
A_9	0.027	0.027	0.040	0.053	0.013	0.027	0.013	0.013
A_{10}	0.027	0.027	0.016	0.053	0.027	0.013	0.027	0.013
A_{11}	0.053	0.053	0.040	0.040	0.040	0.040	0.053	0.067
A_{12}	0.013	0.013	0.016	0.027	0.027	0.013	0.040	0.013
A_{13}	0.027	0.040	0.053	0.053	0.040	0.027	0.027	0.067
A_{14}	0.040	0.067	0.053	0.053	0.040	0.053	0.040	0.067
A_{15}	0.053	0.040	0.040	0.067	0.027	0.027	0.027	0.067

- Step 2: Determination of fuzzy measures
  Considering the order of criteria:
  $C_1 > C_4 > C_2 > C_3 = C_6 = C_8 > C_5 = C_7$
  We have the fuzzy measures to calculate the Choquet integral as:
  μ_1=0.25; μ_{14}=0.84μ_1; μ_{42}=0.49 μ_{14}; μ_{23}=0.9 μ_{42}
  $\mu_{36} = \mu_{68} = \mu_{23}$; $\mu_{85} = \mu_{68}$; $\mu_{57} = \mu_{85}$;

where μ_{ij} are fuzzy measures which are the weights for the different criteria group. We have taken the highest value for μ_1 because criterion 1 is the most important one. The other values are proportional or equal following the criteria order. This weighting is performed in a way such that the sum of all measures is equal to 1.0.

- Step 3: Computation of the Choquet integral
  Table 5 presents the computed values of the Choquet integral.

Table 5: Computed values of the Choquet integral

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Choquet integral
A_1	0.010	0.006	0.004	0.011	0.001	0.003	0.003	0.001	0.041
A_2	0.013	0.005	0.004	0.008	0.001	0.001	0.002	0.001	0.036
A_3	0.010	0.008	0.002	0.008	0.002	0.001	0.001	0.001	0.034
A_4	0.010	0.003	0.004	0.011	0.002	0.001	0.003	0.001	0.037
A_5	0.017	0.008	0.006	0.013	0.003	0.007	0.001	0.007	0.063
A_6	0.007	0.002	0.004	0.011	0.001	0.001	0.001	0.001	0.029
A_7	0.003	0.002	0.002	0.004	0.002	0.001	0.001	0.008	0.022
A_8	0.017	0.002	0.004	0.011	0.001	0.003	0.001	0.007	0.046
A_9	0.007	0.003	0.004	0.011	0.001	0.003	0.001	0.001	0.031
A_{10}	0.007	0.003	0.002	0.011	0.001	0.001	0.001	0.001	0.028
A_{11}	0.013	0.006	0.004	0.008	0.002	0.004	0.003	0.007	0.049
A_{12}	0.003	0.002	0.002	0.005	0.001	0.001	0.002	0.001	0.018
A_{13}	0.007	0.005	0.004	0.011	0.002	0.003	0.001	0.007	0.040
A_{14}	0.010	0.008	0.006	0.011	0.002	0.006	0.002	0.007	0.052
A_{15}	0.010	0.000	0.010	0.010	0.000	0.000	0.000	0.010	0.050

As examples, some of the computed values of the Choquet integral are shown in Table 6.

Table 6: Example computation of the Choquet integral

Alternative	Criteria	
	C_1 - Localization	C_2 – Constructed Area
A_1	0.25 x 0.004 = 0,010	0.12 x 0.05 = 0,006
A_2	0.25 x 0.053 = 0,013	0.12 x 0.040 =0,005

In other words, 0.25 multiply 0.04 equal 0.010 is the product of the fuzzy measure of criteria 1 (with weight equal to 0.25) by the fuzzified value of the utility for alternative A_1 in relation of criteria C_1. Similarly, 0.25 multiply 0.053 equal 0.013 is the product of the fuzzy measure of criteria C_1 (0.25) by the fuzzified value of the utility for alternative A_2 in relation of criteria C_1.

The calculations of the Choquet integral are the sum of all the values obtained for each line of the matrix. For the alternative A_1, we have:

$0.010+0.006+0.004+0.011+0.001+0.003+0.003+0.001=0.041$

Similarly, for the alternative A_2, we obtain:

$0.013+0.005+0.004+0.008+0.001+0.001+0.002+0.001=0.036$

and so on. Thus we obtain Table 7, with values of the Choquet integral for alternatives A_1 and A_2:

Table 7: Values of the Choquet integral for alternatives A_1 and A_2:

Criteria	A_1	A_2
C_1 - Localization	0.010	0.013
C_2 - Constructed Area	0.006	0.005
C_3 - Quality of Construction	0.004	0.004
C_4 - State of Conservation	0.011	0.008
C_5 - Number of garage spaces	0.001	0.001
C_6 - Number of rooms	0.003	0.001
C_7 – Attractions	0.003	0.002
C_8 – Security	0.001	0.001
Choquet integral	0.041	0.036

- Step 4: Ranking of the alternatives

With the values of the Choquet integral we obtain the ranking of the alternatives. This ranking is performed by ordering the obtained values of the Choquet integral. The ranking of the alternatives ordering is shown in Table 8.

Table 8: Ranking of alternatives and values of the Choquet integral

Alternative	Values of the Choquet's integral	Ranking
A_1	0.041	6
A_2	0.036	9
A_3	0.034	10
A_4	0.037	8
A_5	0.063	1
A_6	0.029	12
A_7	0.022	14
A_8	0.046	5
A_9	0.031	11
A_{10}	0.028	13
A_{11}	0.049	4
A_{12}	0.018	15
A_{13}	0.040	7
A_{14}	0.052	2
A_{15}	0.050	3

A comparative analysis of the results is performed by comparing the rank seen in Table 8 with these obtained by using the original TODIM method as in [7]. Table 9 displays the two rankings. The Spearman coefficient of correlation between the two ranks was found equal to 0.9142. This indicates that these two ranks are indeed quite close.

Table 9: Rankings from using Choquet and the original TODIM method

Alternatives	Choquet Ranking	TODIM Ranking	Comparison
A_1	6	5	—
A_2	9	10	—
A_3	10	9	—
A_4	8	7	—
A_5	1	1	Same
A_6	12	11	—
A_7	14	15	—
A_8	5	8	—
A_9	11	14	—
A_{10}	13	12	—
A_{11}	4	3	—

A_{12}	15	13	—
A_{13}	7	4	—
A_{14}	2	2	Same
A_{15}	3	6	—

- Step 5: Sensitivity Analysis

The sensitivity analysis was performed by modifying the fuzzy measures by increasing and decreasing their values and then recalculating the Choquet integral.

The fuzzy measures used in the sensitivity analysis for the Choquet integral were:

$\mu_1 = 0.21$; $\mu_{14} = 0.693\ \mu_1$; $\mu_{42} = 0.93\mu_{14}$; $\mu_{23} = 0.835\ \mu_{42}$

$\mu_{36} = \mu_{68} = \mu_{23}$; $\mu_{85} = 0.75\ \mu_{68}$; $\mu_{57} = \mu_{85}$;

The results from the sensitivity analysis are presented in Table 10.

Table 10: Results from the Sensitivity Analysis

Alternatives	Choquet's initial ranking	Choquet's ranking after sensitivity analysis
A_1	6	6
A_2	9	8
A_3	10	9
A_4	8	7
A_5	1	1
A_6	12	11
A_7	14	13
A_8	5	5
A_9	11	10
A_{10}	13	12
A_{11}	4	3
A_{12}	15	14
A_{13}	7	6
A_{14}	2	2
A_{15}	3	4

4. Conclusions and recommendations for future research

The key conclusions from this case study are listed below: (i) the use of the Choquet integral minimizes the calculations by TODIM since it is unnecessary to normalize the raw data; (ii) not only precise values can be used but also interval data; this second situation would lead to using a fuzzy triangular number; (iii) by using the Choquet integral more complex additive models can be used that allow for taking dependencies between criteria into consideration.

Suggestions for future research follow: (a) extending the TODIM method to situations when input data are not only precise, but also liable to be described by interval or by fuzzy numbers; (b) using the bipolar Choquet integral for taking more complex forms of interdependencies between criteria into consideration as pointed out by [1]; (c) rental values will then be revised, by taking into consideration the existing values, aiming to use the bipolar Choquet integral and then check if the rank will change; (d) making use of either Sugeno's fuzzy inferential system [16] in order to compare the obtained results against these computed by the Choquet-extend TODIM method. In particular, new concepts such as generalizations of Choquet integral [17] as well as the bipolar CPT [18] should be considered.

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